Set-Membership identification of linear systems with input backlash

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Identificazione di sistemi nonlineari

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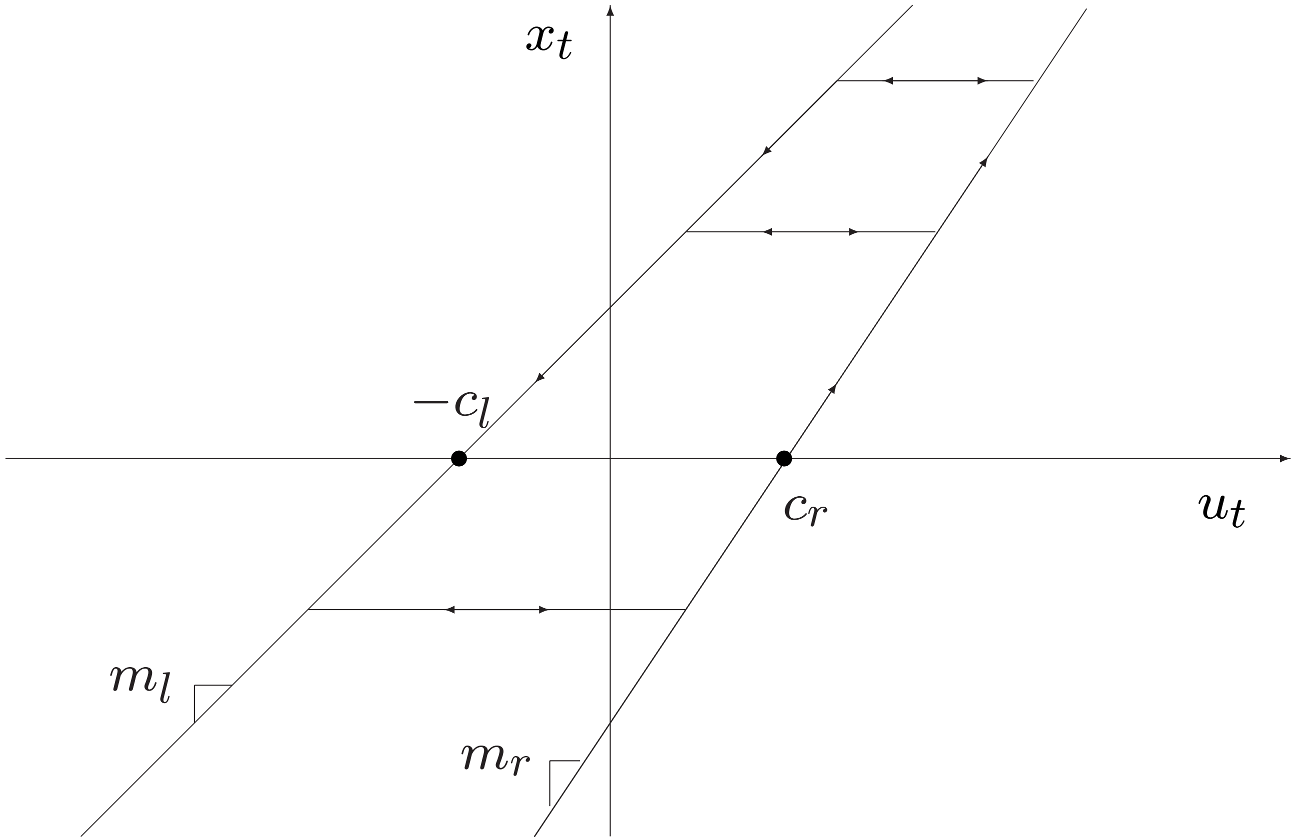
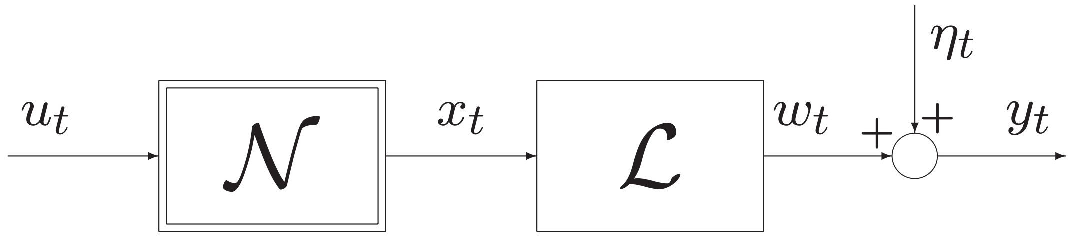
Backlash nonlinearity

- Backlash is a hard (i.e. non-differentiable) and dynamic (i.e. with memory) nonlinearity.
- Backlash is a typical characteristic of mechanical connections (e.g. gearboxes).



Linear systems with input backlash

- The structure resemble that of Hammerstein systems
- Backlash nonlinearity is considered instead of the usual memoryless nonlinearity



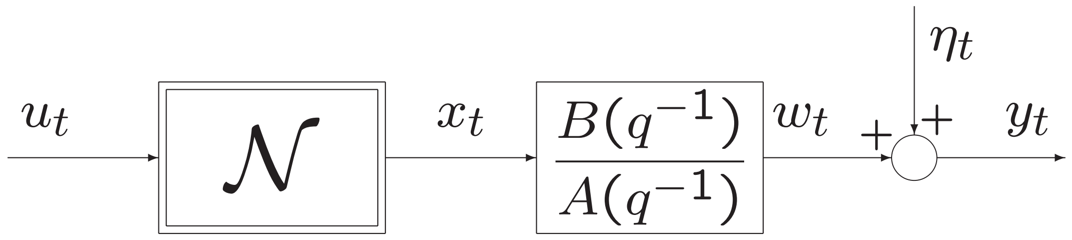
where:

$\mathcal{N}$: backlash nonlinearity
$\mathcal{L}$: linear subsystem
x_t: inner signal not measurable

Identification of linear systems with input backlash

- Motivations:
  - Backlash severely limits the performance of control systems (may cause delays, oscillations, limit cycles, inaccuracy, ...)
  - Adaptive and/or robust control techniques can cope with such limitations.
  - Robust control techniques need bounds on the parameters of uncertain backlash
    (see, e.g., **M. Corradini et al.,"A VSC approach for the robust stabilization of nonlinear plants with uncertain non-smooth actuator nonlinearities — A unified framework"**, IEEE TAC 2004)
- Identification approach:
  - Statistical framework:
    E.W. Bai ,"Identification of linear system with hard input nonlinearity", *Automatica 2002*
  - Set-membership framework: focus of this lesson

Problem formulation



$$x_t = \begin{cases} m_l(u_t + c_l) & \text{for } u_t \leq z_l \\ m_r(u_t - c_r) & \text{for } u_t \geq z_r \\ x_{t-1} & \text{for } z_l < u_t < z_r \end{cases}$$

$$m_l > 0,\ m_r > 0,\ c_l > 0,\ c_r > 0$$

$$z_l = \frac{x_{t-1}}{m_l} - c_l, \qquad z_r = \frac{x_{t-1}}{m_r} + c_r$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}$$

$$q^{-1} w_t = w_{t-1}$$

Problem formulation

- **Aim:** compute bounds on the parameters $\gamma^\top = [m_l \ c_l \ m_r \ c_r]$ and $\theta^\top = [a_1 \ \dots \ a_{na} \ b_0 \ \dots \ b_{nb}]$.

- **Prior assumption on the system:**

1. stability;
2. na and nb are known;
3. the steady-state gain is not zero;
4. a rough upper bound on the settling time of the system is known;

- **Prior assumption on the measurement uncertainty:**

1. $\{\eta_t\}$ is UBB: $\|\{\eta_t\}\|_\infty \leq \Delta\eta_t$;
2. $\Delta\eta_t$ is known;

Proposed solution: preliminary

Three-stage solution:

- First stage: computation of bounds on the backlash parameters γ.
- Second stage: computation of bounds on the inner (unmeasurable) signal x_t.
- Third stage: computation of bounds on the linear block parameters θ.

Proposed solution: preliminary

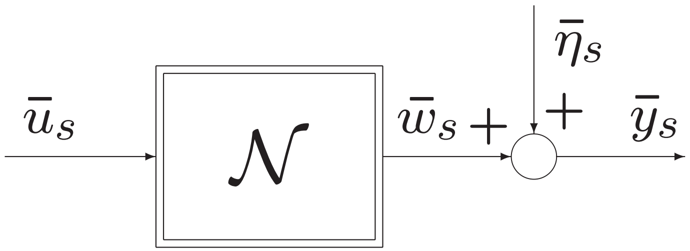
Remark 1: The parameterization is not unique

$\Rightarrow$ assume $g_{dc} = \dfrac{\sum_{j=0}^{nb} b_j}{1 + \sum_{i=1}^{na} a_i} = 1$

Remark 2: Stimulate the system with a set of square wave inputs with different amplitudes

$$\Downarrow$$

Steady-state operating conditions:



Proposed solution: first-stage

1) Bounds on the backlash parameters

- Get $M \geq 2$ *steady-state* measurements:

$$\bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, \quad \text{for } \bar{u}_i \geq \frac{\bar{x}_{i-1}}{m_r} + c_r \quad i = 1, \ldots, M$$

$$\bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, \quad \text{for } \bar{u}_j \leq \frac{\bar{x}_{j-1}}{m_l} - c_l \quad j = 1, \ldots, M$$

- The *feasible parameter set* of the backlash, is defined by the following equations:

$$\mathcal{D}_\gamma = \mathcal{D}_\gamma^r \bigcup \mathcal{D}_\gamma^l$$

$$\mathcal{D}_\gamma^r = \{m_r, c_r \in R^+ : \bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, |\ \bar{\eta}_i\ | \leq \Delta\bar{\eta}_i; \quad i = 1, \ldots, M\} \quad (1)$$

$$\mathcal{D}_\gamma^l = \{m_l, c_l \in R^+ : \bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, |\ \bar{\eta}_j\ | \leq \Delta\bar{\eta}_j; \quad j = 1, \ldots, M\} \quad (2)$$

Proposed solution: preliminary

Remarks:

- $\mathcal{D}_\gamma = \mathcal{D}_\gamma^r \bigcup \mathcal{D}_\gamma^l$ exactly described by the following nonlinear constraints

$$\bar{y}_i - m_r(\bar{u}_i - c_r) \geq -\Delta\bar{\eta}_i, \quad \bar{y}_i - m_r(\bar{u}_i - c_r) \leq \Delta\bar{\eta}_i, m_r > 0, c_r > 0, i = 1, \ldots, M$$

$$\bar{y}_j - m_l(\bar{u}_j + c_l) \geq -\Delta\bar{\eta}_j, \quad \bar{y}_j - m_l(\bar{u}_j + c_l) \leq \Delta\bar{\eta}_j, m_l > 0, c_l > 0, j = 1, \ldots, M$$

- $\mathcal{D}_\gamma^l$ and $\mathcal{D}_\gamma^r$ are disjoint sets (they can be handled separately).

- $\mathcal{D}_\gamma^l$ and $\mathcal{D}_\gamma^r$ have the same mathematical structure (enjoy the same properties).

- Results derived for $\mathcal{D}_\gamma^r$ in the following slides, are also applicable to $\mathcal{D}_\gamma^l$.

Proposed solution: first-stage

Definition 1 (Constraints boundaries defining $\mathcal{D}_\gamma^r$)
the constraints defining the $\mathcal{D}_\gamma$ corresponding to the s-th sets of data $(\overline{u}_s, \overline{y}_s)$ are:

$$
\begin{aligned}
h_r^+(\overline{u}_s) &\doteq \{m_r \in R^+, c_r \in R^+ : \overline{y}_s + \Delta\eta_s = m_r(\overline{u}_s - c_r)\} \\
h_r^-(\overline{u}_s) &\doteq \{m_r \in R^+, c_r \in R^+ : \overline{y}_s - \Delta\eta_s = m_r(\overline{u}_s - c_r)\}
\end{aligned}
$$

Definition 2 (Boundary of $\mathcal{D}_\gamma^r$)

$$\text{Boundary of } \mathcal{D}_\gamma^r \doteq \mathcal{H}(\mathcal{D}_\gamma^r)$$

Definition 3 (Edges of $\mathcal{D}_\gamma^r$)

$$\tilde{h}_r^+(\overline{u}_s) \doteq h_r^+(\overline{u}_s) \bigcap \mathcal{D}_\gamma^r = \{\, m_r, c_r \in \mathcal{D}_\gamma^r : \overline{y}_s + \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$$

$$\tilde{h}_r^-(\overline{u}_s) \doteq h_r^-(\overline{u}_s) \bigcap \mathcal{D}_\gamma^r = \{\, m_r, c_r \in \mathcal{D}_\gamma^r : \overline{y}_s - \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$$

Proposed solution: first-stage

Definition 4 (Constraints intersections)

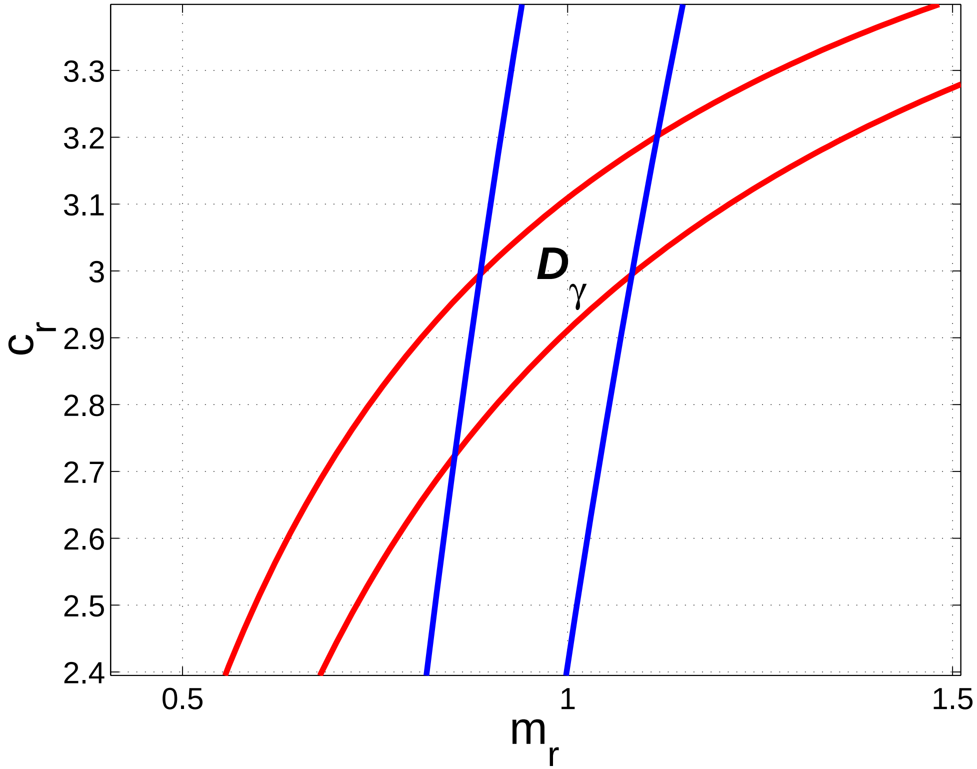
$$\mathcal{I}_{\gamma}^{r} = \{(m_r, c_r) \in R^2 : \{h_r^{+}(\overline{u}_i), h_r^{-}(\overline{u}_i)\} \bigcap \{h_r^{+}(\overline{u}_j), h_r^{-}(\overline{u}_j)\} \neq \emptyset; i,j = 1, \ldots, M; i \neq j\}$$

Definition 5 (Vertices of $\mathcal{D}_{\gamma}^{r}$)

$$\mathcal{V}(\mathcal{D}_{\gamma}^{r}) = \mathcal{I}_{\gamma}^{r} \bigcap \mathcal{D}_{\gamma}^{r}.$$

Remarks:

- An exact description of $\mathcal{D}_{\gamma}^{r}$ can be given in terms of edges and vertices.
- A recursive algorithm for the computation of edges and vertices of $\mathcal{D}_{\gamma}^{r}$ has been provided.



Proposed solution: first-stage

Tights bounds on $\gamma_1 = m_r$ and $\gamma_2 = c_r$ obtained computing the orthotope outer-bound:

$$\mathcal{B}^r_\gamma = \{\gamma \in R^2 : \gamma_j = \gamma^c_j + \delta\gamma_j, |\ \delta\gamma_j\ | \leq \Delta\gamma_j, j = 1, 2\}$$

$$\gamma_j{}^c = \frac{\gamma_j^{min} + \gamma_j^{max}}{2}, \qquad \Delta\gamma_j = \frac{|\ \gamma_j^{max} - \gamma_j^{min}\ |}{2}$$

$$\gamma_j^{min} = \min_{\gamma \in \mathcal{D}^r_\gamma} \gamma_j, \quad \gamma_j^{max} = \max_{\gamma \in \mathcal{D}^r_\gamma} \gamma_j. \tag{3}$$

- $\mathcal{B}_\gamma$ is obtained solving problems (3) which are 2 nonconvex optimization problems with 2 variables and $2M$ constraints.
- **<u>Main Result 1</u>:**
  The global optimal solutions of problems (3) occur on the vertices of $\mathcal{D}^r_\gamma$.

Proposed solution: second-stage

2) Bounds on the inner signal x_t

Result 1

The input of the backlash u_t is a PRBS with levels $\pm u^{\star}$, $u^{\star} > c_r$, $-u^{\star} > c_l$

$$\Downarrow$$

The output of the backlash x_t is a PRBS with levels $\bar{x}^{\star} = m_r(u^{\star} - c_r)$, $\underline{x}^{\star} = m_l(u^{\star} - c_l)$.

- Bounds on the inner signal level $x^{\star}$ are computed as:

$$\bar{x}^{\star min} = \min_{m_r, c_r \in \mathcal{D}_{\gamma}^{r}} m_r(u^{\star} - c_r), \quad \text{for } u^{\star} \geq c_r \tag{4}$$

$$\bar{x}^{\star max} = \max_{m_r, c_r \in \mathcal{D}_{\gamma}^{r}} m_r(u^{\star} - c_r), \quad \text{for } u^{\star} \geq c_r \tag{5}$$

Proposed solution: second-stage

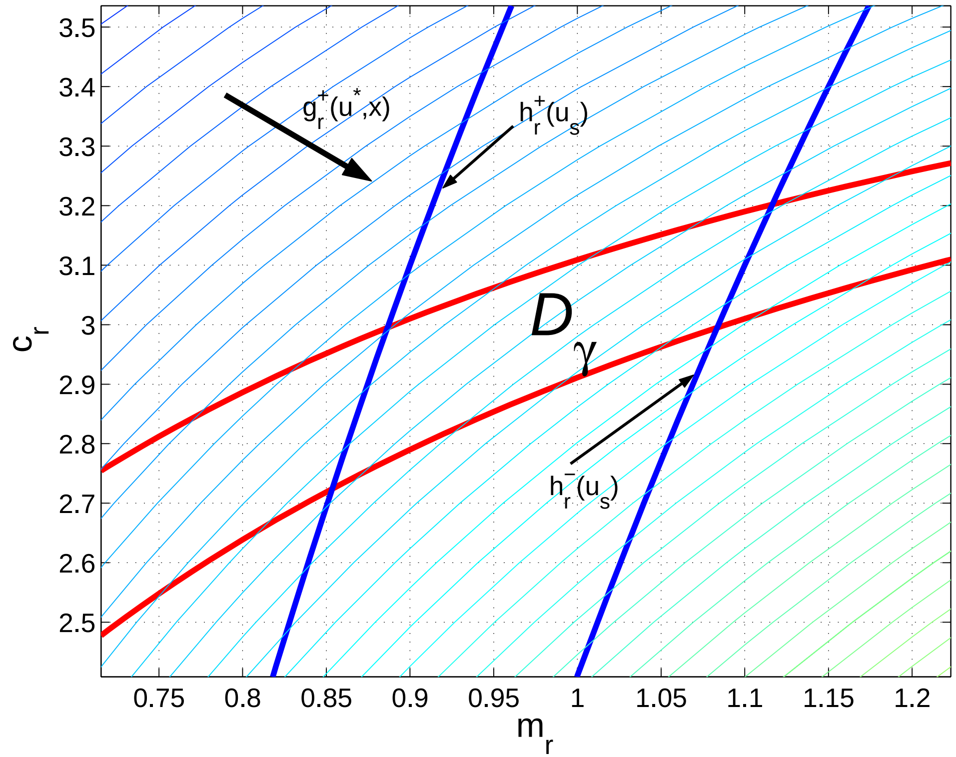
Definition 6 (x-level curve of the objective function to be optimized)

$$g_r(u^\star, x) \doteq \{m_r \in R^+, c_r \in R^+ : x = m_r(u^\star - c_r)\}$$

Main Result 2

(Computation of the inner signal bounds)

The global optimal solutions of problems (4) and (5) occur on the vertices of $\mathcal{D}_\gamma^r$.



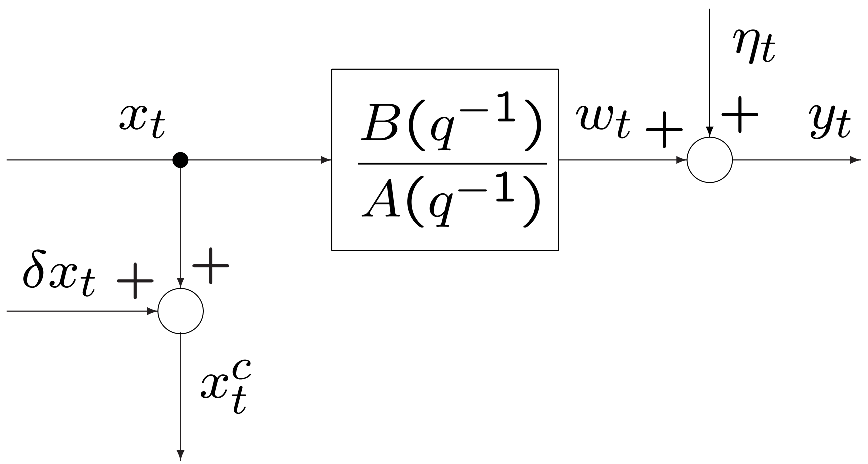
Proposed solution: third-stage

3) Bounds on the linear block parameters

The input PRBS sequence $\{u_t\}$ of levels $\pm u^\star$ is applied and the (noisy) output sequence $\{y_t\}$ is measured.

$$\Downarrow$$

Errors-in-variables (EIV) problem with UBB errors



$$x_t^c = \frac{\overline{x}^{\star min} + \overline{x}^{\star max}}{2}$$

$$\mid \delta x_t \mid \leq \Delta x_t$$

$$\Delta x_t = \frac{\overline{x}^{\star max} - \overline{x}^{\star min}}{2}$$

Proposed solution: EIV problem

Exploiting previous results on static EIV problem with bounded errors

(**V. Cerone,"Feasible parameter set of linear models with bounded errors in all variables"**, *Automatica 1993*)

$$\Downarrow$$

a polytopic outer approximation $\mathcal{D}'_\theta$ of the *feasible parameter set* $\mathcal{D}_\theta$ is characterized by:

$$(\phi_t - \Delta\phi_t)^\top \theta \leq y_t + \Delta\eta_t, \quad (\phi_t + \Delta\phi_t)^\top \theta \geq y_t - \Delta\eta_t$$

$$\phi_t^\top = \left[-y_{t-1} \ldots - y_{t-na} \; x_t^c \; x_{t-1}^c \ldots x_{t-nb}^c \right]$$

$$\Delta\phi_t^\top = [\Delta\eta_{t-1}\text{sgn}(a_1) \quad \ldots \quad \Delta\eta_{t-na}\text{sgn}(a_{na})$$

$$\Delta x_t \text{sgn}(b_0) \quad \Delta x_{t-1}\text{sgn}(b_1) \quad \ldots \quad \Delta x_{t-nb}\text{sgn}(b_{nb})]$$

$$[1 \quad \ldots \quad 1 \quad -1 \quad -1 \quad \ldots \quad -1]\,\theta = -1$$

Proposed solution: EIV problem

Parameter uncertainty intervals $\Delta\theta_j$ are provided by the bounding orthotope $\mathcal{B}_\theta$:

$$\mathcal{B}_\theta = \{\theta \in R^p : \theta_j = \theta_j^c + \delta\theta_j, \mid \delta\theta_j \mid \leq \Delta\theta_j/2, j = 1, \ldots, p\},$$

$$\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2},$$

$$\Delta\theta_j = \mid \theta_j^{max} - \theta_j^{min} \mid,$$

$$\theta_j^{min} = \min_{\theta \in \mathcal{D}'_\theta} \theta_j, \quad \theta_j^{max} = \max_{\theta \in \mathcal{D}'_\theta} \theta_j.$$

Computational aspects:

- θ_j^{min} and θ_j^{max} are computed by means of linear programming techniques.

Example:

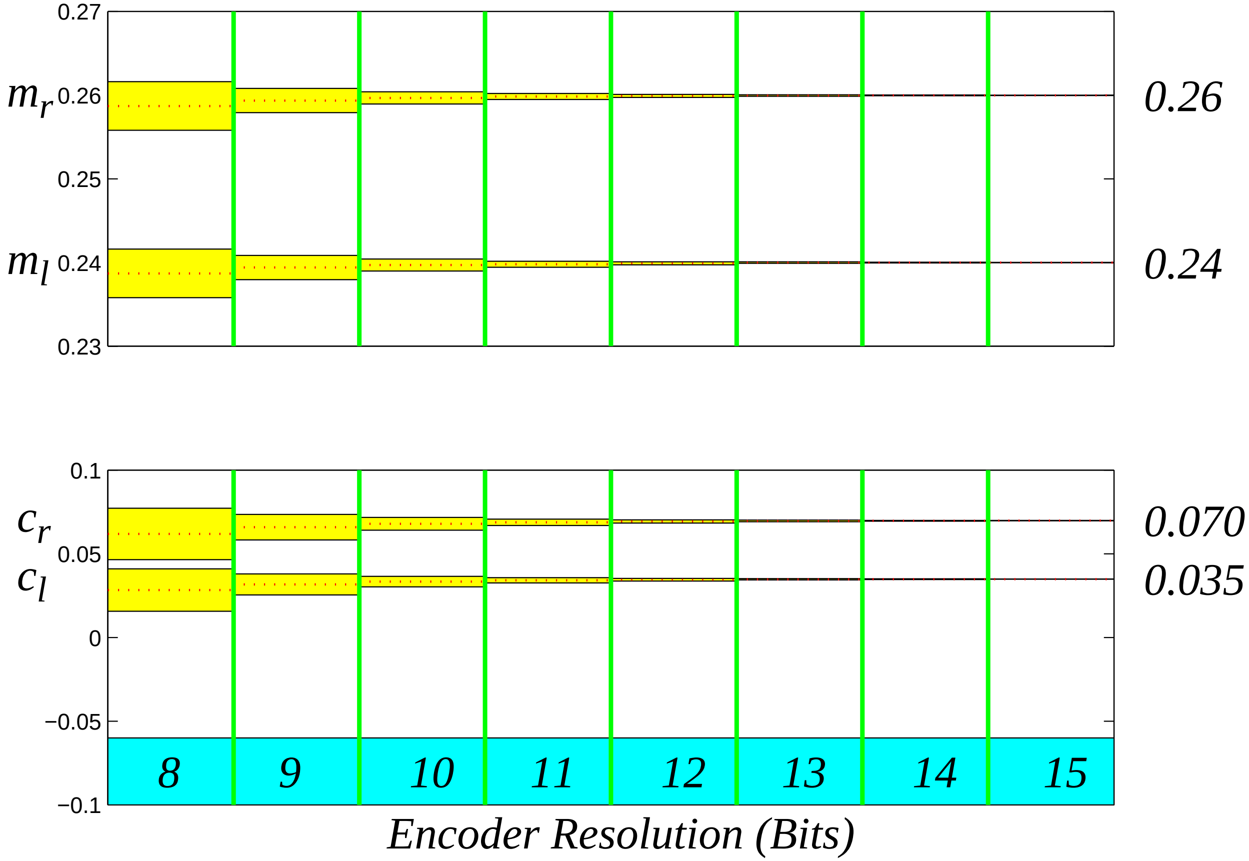
Parameters of the simulated system:
$m_l = 0.24$, $m_r = 0.26$, $c_l = 0.035$, $c_r = 0.070$;
$A(q^{-1}) = (1 - 0.5q^{-1} - 0.1q^{-2})$;
$B(q^{-1}) = (0.2q^{-1} + 1.2q^{-2})$

Measurement output errors:
Bounded absolute output errors have been considered:

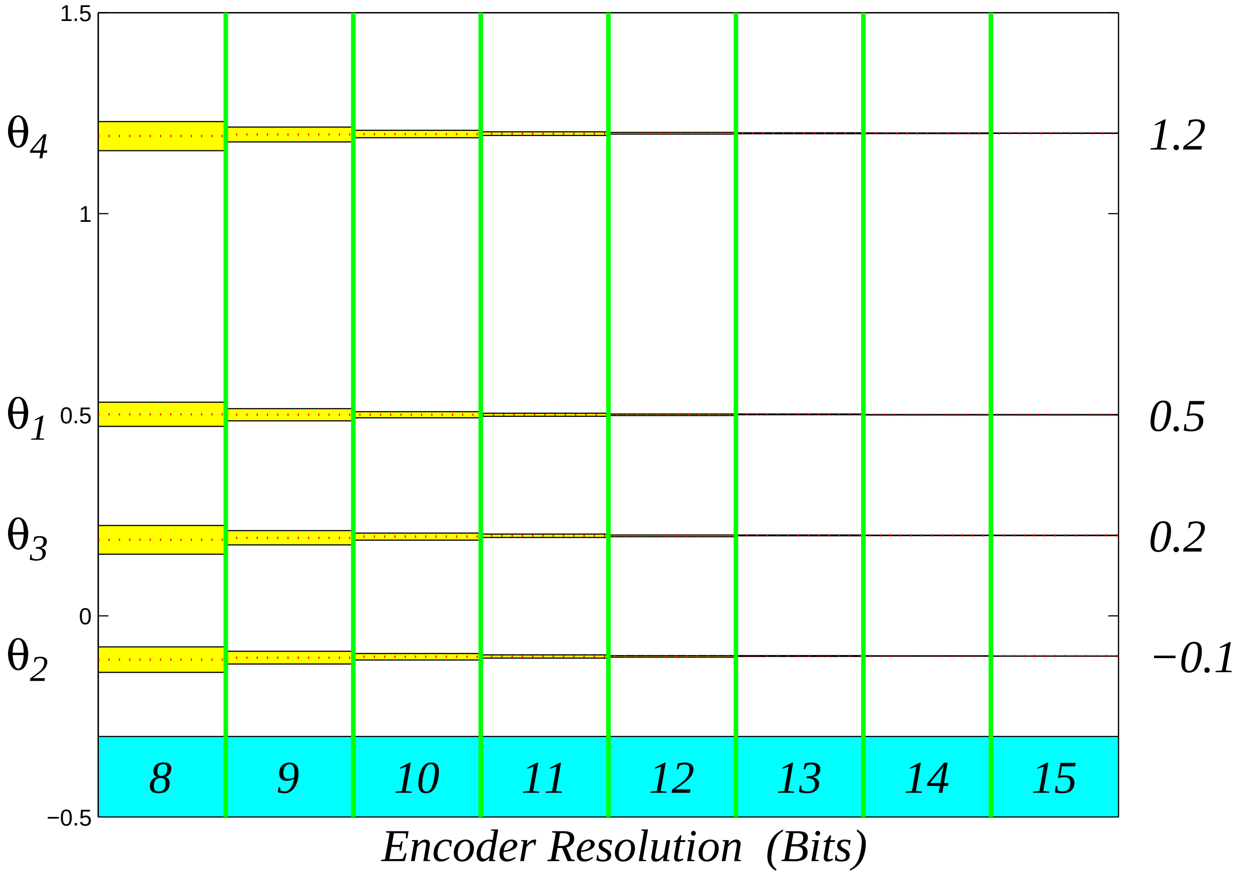
$\mid \eta_t \mid \leq \Delta\eta_t$; $\{\eta_t\}$ belongs to the uniform distribution $U[-\Delta\eta_t, +\Delta\eta_t]$.
$\mid \bar{\eta}_s \mid \leq \Delta\bar{\eta}_s$; $\{\bar{\eta}_s\}$ belongs to the uniform distribution $U[-\Delta\bar{\eta}_s, +\Delta\bar{\eta}_s]$

Eight different values of $\Delta\eta = \Delta\eta_t = \Delta\bar{\eta}_s$ were chosen in such a way to simulate the errors of eight commercial absolute binary encoder with number of bits n_{bits} varying from 8 to 15.

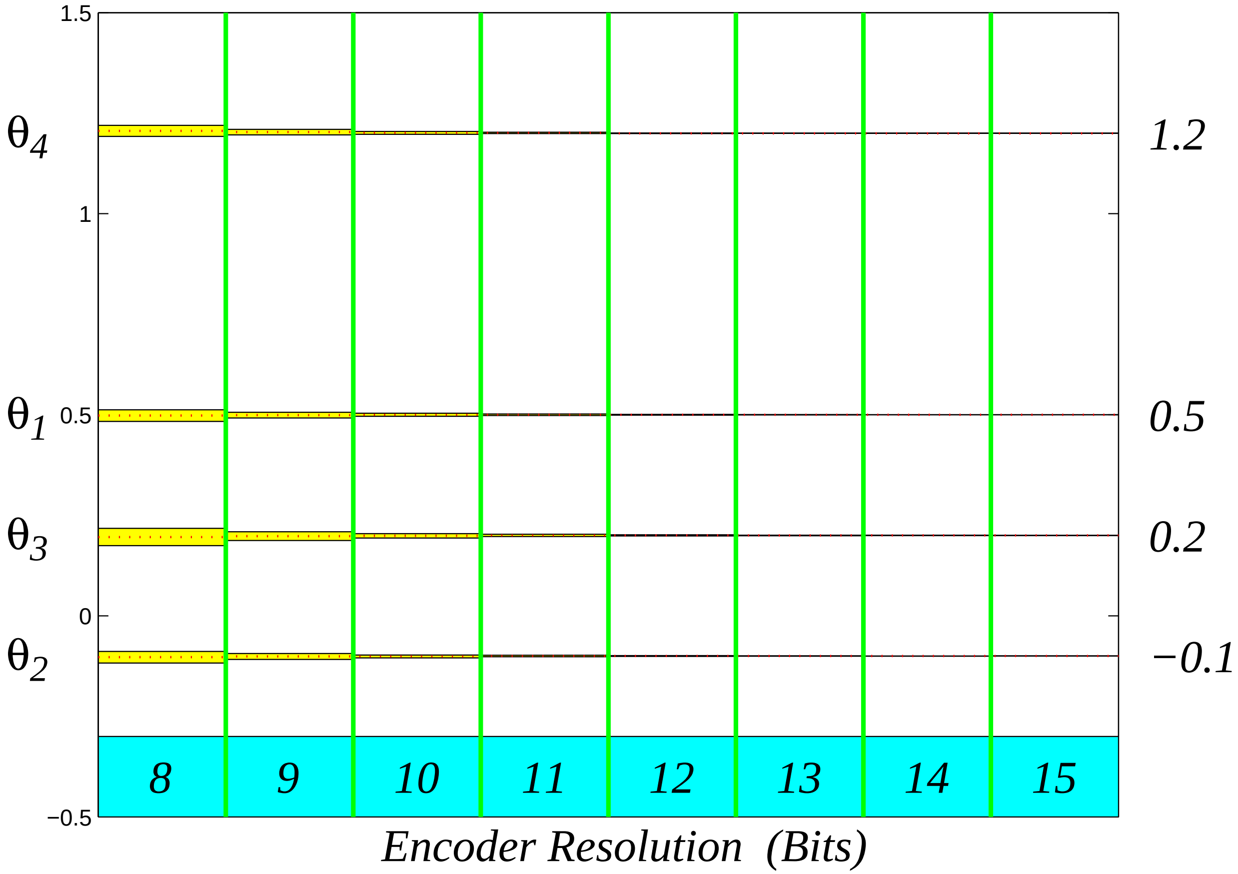
Backlash parameters: central estimates and parameters uncertainty bounds (M = 50)



Linear block parameters: central estimates and parameters uncertainty bounds (N = 100)



Linear block parameters: central estimates and parameters uncertainty bounds (N = 1000)



Conclusions

- The proposed three-stage parameter bounding procedure provides:
  - tight bounds on the parameters of the backlash using steady-state input-output data;
  - overbounds on the parameters of the linear part, through the computation of tight bounds on the unmeasurable inner signal x_t;
- The numerical example has showed the effectiveness of the proposed procedure.
- The approach is computationally tractable: the computation related to the above example (M=2, N=[100,1000]) required few seconds on a standard notebook (AMD 3200)

~~Reference paper~~

References

Motivating papers

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Identification

- E. Bai, “Identification of linear systems with hard input nonlinearities of known structure,” Automatica, vol. 38, pp. 853–860, 2002.
- V. Cerone, D. Regruto, “Bounding the Parameters of Linear Systems With Input Backlash,” IEEE Trans. Autom. Control, vol. 52, no. 3, pp. 531–536, March 2007.
- V. Cerone, “Feasible parameter set for linear models with bounded errors in all variable,” Automatica, vol. 29, no. 6, pp. 1551–1555, 1993.